Aggregate Multi-commodity Stochastic Models for Collaborative Trajectory Options Program (CTOP)

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1. General Background of CTOP and Problem Under Investigation

2. A Brief Literature Review

3. Two-stage Static Model

4. Multistage Semi-Dynamic Model

5. Multistage Dynamic Model

6. Numerical Results

7. Conclusions
Classical Traffic Management Initiatives (TMIs)

(a) Demand and Capacity at a GDP Airport

- GDP is a terminal TMI that flights are delayed at their departure airports in order to reconcile demand with capacity at arrival airports.

(b) FCA Controls En Route Traffic in an AFP

- AFP is a TMI that identifies constraints in the en route domain and meter the demand through Flow Constrained Area (FCA).

- Flights sometimes may need to move away from an area of airspace due to weather or high traffic volume. A reroute is used to replaced the filed route in flight plan.

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Collaborative Trajectory Options Program (CTOP)

- CTOP is a TMI that automatically assigns delays and/or reroutes around one or more FCA-based airspace constraints to balance demand with capacity.
- A CTOP allows operators to submit a set of desired reroute options (called a Trajectory Options Set or TOS).

### Flight ID

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### Trajectory Option Set

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**Figure 3:** TOS Example of a Flight from LAX to ATL
CTOP Introduction Ctd.

The assigned route is the one that minimizes Adjusted Cost:

Adjusted Cost = RTC + Required Ground Delay

route 1: 70=0 + 70
route 2: 50=30 + 20
route 3: 60=50 + 10
route 4: 70=60 + 10
route 5: 70=70 + 0

Figure 4: Flight Routes in the TOS and Location of FCAs

- CTOP has the ability to handle multiple FCAs with a single program, allowing different parts of the program to be adjusted independently as conditions change.
This paper endeavors to answer the following FAA side research question: given demand estimation, how can we best manage this demand through multiple congested airspace regions with stochastic capacities.

1. Our objective is to minimize system delay costs. For equality and computation reasons, aggregate models are preferred.
2. We assume for each flight which route it will take, therefore the demand for congested regions at each time period is known.
3. Traffic flows need to be managed in an integrated way.
4. We need to deal with a decision making under uncertainty problem.

CTOP is a TMI that handles multiple constrained resources and is thus closely related with the above research question.

1. We will show the above research problem can be viewed a subproblem in CTOP rate optimization.
A Brief Literature Review

- Single Airport Ground Hold Problem has been a classical research problem
- Simulation-based optimization, Markov Decision Process (MDP), Chance-constrained optimization, Stochastic optimization, etc
- Ball’s (2003) two-stage static model
  - Using scenario information, it can optimize the departure times of all flights when a Ground Delay Program is just proposed
  - A flight can be only ground delayed once (highest predictability)
- Richetta’s (1994) Multistage semi-dynamic model
  - Using scenario tree structure and flight departure information, determine the departure time of a flight at its scheduled departure time
  - A flight can be only ground delayed once (medium predictability/cost)
- Mukherjee’s (2007) Multistage dynamic model
  - Using scenario tree structure, flight departure and enroute information, dynamically ground delay non-departed flights
  - A flight can be ground delayed multiply times (lowest system cost)
Scenarios and Scenario Tree

- In the stochastic programming framework, a scenario is a possible realization of random variables.

- In air traffic management problem, airport and airspace capacity uncertainty is represented by a finite number of scenarios arranged in a scenario tree. A branch point is the time we acquire new information and similar scenarios evolve into distinct scenarios.

Figure 5: Scenario Tree
Flow Constrained Area (FCA) which is an artificial line or region in the airspace and serves like a valve to control the traffic flows into a region.

Potentially Constrained Area (PCA), which is the actual troubled area and whose future capacity realization is represented by a scenario tree.

The FCA rates is a fixed vector which is not scenario dependent. The PCA rate is a random vector.

In this work, we will focused on PCA rate optimization.
Model Assumptions

1. We assume the flights captured by CTOP are not controlled by other TMI at the same time (no TMI interaction)

2. We assume the assigned route information, the topology of PCA network, unimpeded PCA entry times, CTOP window, and scenario-based capacity information are all available (model inputs)

3. We assume all flights are required to exit the PCA network by the end of the planning horizon (boundary condition)

4. We assume we can only delay a flight by integer multiples of the size of a time period. In this paper, we use the bin size of 15 minutes (interval-based model)

5. We assume we can ground hold and air hold flights (centralized idealized model, theoretic lower bound)
Why Multi-commodity Flow Model

- GDP models or single airport rerouting model are essentially single commodity flow models, since all traffic is bound for a single airport
- CTOP planning is in nature a multi-commodity problem, since flights will traverse different congested airspace and reach different destinations

[Avijit Mukherjee PhD Thesis]
In the following stochastic models, flights are grouped by Path, which is the sequence of PCAs flights traverse. A path uniquely defines a commodity in a multi-commodity flow model, e.g.

1. Flights directly fly to ERW
2. Flights pass PCA1 and then fly to EWR

We call the demand directly from airports the Direct Demand, the demand from the upstream PCA Upstream Demand.

For a specific commodity, direct demand can be from multiple airports, the upstream demand can be only from one upstream PCA.

Ground delay can only be applied to direct demand.
Two-stage Static Model Constraints

- Relationship between Planned Acceptance Rate, Scheduled Direct Demand and Ground Delay

\[ P_{t,r}^k = D_{t,r}^k - (G_{t,r}^k - G_{t-1,r}^k) \quad k = r_1, \forall r, t \]

1. \( P_{t,r}^k \): Planned direct demand at PCA \( k \) at time period \( t \) from flights with same path \( r \) (primary decision variables)
2. \( D_{t,r}^k \): Scheduled direct demand at PCA \( k \) at time period \( t \) from flights with same path \( r \)
3. \( G_{t,r}^k \): Number of flights with same path \( r \) whose arrival time at PCA \( k \) is adjusted from time interval \( t \) to \( t + 1 \) or later using ground delay

- For commodity \( r \), we have this constraint at the first PCA on path \( r \), \( k = r_1 \)
Two-stage Static Model Constraints Cont.

- Relationship between Planned Acceptance Rate, Flights Actually Pass PCA and Air Delay

\[
L^k_{t,r,q} = \begin{cases} 
  P^k_{t,r} - (A^k_{t,r,q} - A^k_{t-1,r,q}) & \text{if } k = r_1 \\
  \text{UpPCA}^k_{t,r,q} - (A^k_{t,r,q} - A^k_{t-1,r,q}) & \text{else}
\end{cases}
\]

\[
\text{UpPCA}^k_{t,r,q} = L^{k'}_{t-\Delta^{k',k},r,q} \quad (k', k) \in r
\]

1. \(L^k_{t,r,q}\) Number of flights with same path \(r\) that actually pass PCA \(k\) during time \(t\) in scenario \(q\)
2. \(\Delta^{k',k}\) Number of time periods to travel from upstream PCA \(k'\) to \(k\)

- Number of flights which can actually pass PCA \(k\) is scenario dependent
- For commodity \(r\), we have this constraint at all the PCAs along path \(r\)
In GDP models, we often impose the following boundary conditions to ensure all flights will land at the end of planning horizon:

\[ G_{T+1} = A_{T+1,q} = 0 \]

Boundary conditions can guarantee all GDP flights are properly handled, so that we can fairly compare the performance of different models.

At the end of a GDP, if for model 1 there are several flights still on the ground or just depart, but for model 2 all flights have landed/exited. Even though model 1 may have a smaller objective value, we cannot say model 1 is superior to model 2.
In CTOP where there are multiple constrained resources, constraints $G_{T+1} = A_{T+1,q} = 0$ are not sufficient to ensure all flights will land/exit at the end of planning horizon. Because there may still be en route flights between PCAs $L_{t,r,q}^k$

We will need to explicitly enforce that, for each commodity and for each scenario, the total demand of commodity $r$ equals to the cumulative amount of commodity $r$ which exits the PCA system via the last PCA on path $r$

$$\sum_{t=1}^{T} D_{t,r}^{k=r_1} = \sum_{t=1}^{T} L_{t,r,q}^{k=r-1} \quad \forall r, q$$
The objective function minimizes the ground delay and expected air delay cost

$$\min c_g \sum_{t=1}^{T} \sum_{r} G_{t,r}^{k=r_1} + c_a \sum_{q=1}^{Q} p_q \sum_{k \in \text{PCAs}} \sum_{t=1}^{T} \sum_{r} A_{t,r,q}^k$$

1. For flights belong to commodity $r$, ground delay can only happen before the first PCA on path $r$, air delay can happen at any PCA along path $r$.
2. Like Ball’s model, ground delay decisions are made at the beginning of the planning horizon. The air delays are scenario dependent.

Important capacity constraint

$$M_{t,q}^k \geq \sum_r L_{t,r,q}^k \quad \forall t, q, k$$

We require all the decision variables to be nonnegative

$$D_{t,r}^k, P_{t,r}^k, G_{t,r}^k, L_{t,r,q}^k \geq 0 \quad \forall t, r, q, k$$
A drawback of the static model is that we do not take advantage of the updated weather information or the structure of a scenario tree. The semi-dynamic model which could partially overcome this limitation we will use the concept of stage. A stage can comprise several time periods, at which we have the same weather information.

Figure 7: Scenario Tree
Multistage Semi-Dynamic Model Constraints

- Planned Acceptance Rate and Scheduled Direct Demand satisfies

\[
\sum_{t'=t}^{T} X_{s,t,t',r}^{k,q} = S_{s,t,r}^{k} \quad \forall s, \forall q, \forall r
\]

\[
P_{t,r}^{k,q} = \sum_{s} \sum_{t' \leq t} X_{s,t,t',r}^{k,q} \quad \forall q, \forall r
\]

1. $S_{s,t,r}^{k}$ Number of flights with same path $r$ originally scheduled to depart in stage $s$ and arrive in PCA $k$ in interval $t$

2. $X_{s,t,t',r}^{k,q}$ Number of flights with same path $r$, originally scheduled to depart in stage $s$ arrive in PCA $k$ in interval $t$, rescheduled to arrive in interval $t'$ under scenario $q$ (primary decision variables)

3. $P_{t,r}^{k,q}$ Planned direct demand at PCA $k$ in time interval $t$ from flights with same path $r$ in scenario $q$ (now scenario dependent)

- The ground delay decisions are not all made at the beginning and at the same time. We wait until a flight’ originally scheduled departure time (the beginning of corresponding stage, to be exact) and since more weather information is available, we can make better ground decisions
The air delay constraints are similar to the constraints in static model

\[ L_{t,r,q}^k = \begin{cases} P_{t,r,q}^k - (A_{t,r,q}^k - A_{t-1,r,q}^k) & \text{if } k = r_1 \\ \text{UpPCA}_t^k - (A_{t,r,q}^k - A_{t-1,r,q}^k) & \text{else} \end{cases} \]

\[ \text{UpPCA}_t^k = L_{t-\Delta_{t,r,q}}^{k',r,q} \quad (k', k) \in r \]

We now have nonanticipativity constraints, which ensure that decisions made at time \( t \) are solely based on the weather information available at that time

\[ X_{s,t,t',r}^{k,q_1} = \cdots = X_{s,t,t',r}^{k,q_{N_b}} \]
The objective function minimizes the expected ground and air delay cost

\[
\min \sum_{q=1}^{Q} p_q \left\{ \sum_{s} \sum_{t'=t}^{T} \sum_{r} c_g (t' - t) X_{s,t,t',r}^{k,q} + \sum_{k \in \text{PCAs}} \sum_{t=1}^{T} \sum_{r} c_a A_{t,r,q}^k \right\}
\]

The capacity constraints, non-negativity constraints, boundary conditions are very similar to static model

\[
M_{t,q}^k \geq \sum_r L_{t,r,q}^k \quad \forall t, q, k
\]
\[
0 \leq X_{s,t,t',r}^{k,q}, P_{t,r}^k, G_{t,r}^k, L_{t,r,q}^k
\]
\[
G_0^k = A_0, r = 0
\]
\[
\sum_s \sum_t S_{s,t,r}^{k=r_1} = \sum_{t=1}^{T} L_{t,r,q}^{k=r-1} \quad \forall r, q
\]
Multistage Dynamic Model

- We know that as long as a flight is still on the ground, in theory we can further ground hold it.
- In dynamic model, when making ground delay decisions, we will consider the fact this flight may be further ground delayed later on (“plan to replan”)
- **Planned Release Rate**, **Scheduled Departure Demand** and **Ground Delay at departing airports now** satisfy:

  $P_{t,L,r}^k = D_{t,L,r}^k - (G_{t,L,r}^k - G_{t-1,L,r}^k) \quad \forall t, L, r$

- $P_{t,L,r}^k$ is the number of flights with the same path $r$, flight time $L$ (to the first PCA $k$) and departure time $t$ in scenario $q$ (primary decision variables).
- One major difference between dynamic model and previous two models is that we will also use a flight’s en route time information.
- This is because we enforce the nonanticipativity constraints at a flight’s actual departure time, we need to know if we let these flights take off now, how long it will reach the PCA and become real demand.
Objective function

\[
\min \sum_{q=1}^{Q} p_q \left\{ c_g \sum_{t=1}^{T} \sum_{L} \sum_{r} G_{t,L,r}^{k=r_1,q} + c_a \sum_{k \in \text{PCAs}} \sum_{t=1}^{T} \sum_{r} A_{t,r,q}^{k} \right\}
\]

Nonanticipativity constraints for the number of flights which are planned to release at time \( t \) for all groups

\[
P_{t,L,r}^{k,q_1^b} = \ldots = P_{t,L,r}^{k,q_N^b}
\]

The air delay constraints

\[
L_{t,r,q}^{k} = \begin{cases} 
\sum_{L} P_{t-L,L,r}^{k,q} - (A_{t,r,q}^{k} - A_{t-1,r,q}^{k}) & \text{if } k = r_1 \\
\upPCA_{t,r,q}^{k} - (A_{t,r,q}^{k} - A_{t-1,r,q}^{k}) & \text{else}
\end{cases}
\]

\[
\upPCA_{t,r,q}^{k} = L_{t-\Delta_{k',k},r,q}^{k'} (k', k) \in r
\]

If a flight is released at \( t - L \) and its en route time is \( L \), it will arrive at its first PCA at \( t \).
For two-stage static models, after we solving the model, we will know the planned demand to each PCA $k \ P^k_{t,r}$. This provides us a reference for setting the FCA rate $\sum_r P^k_{t,r}$ if that FCA is located before PCA $k$.

This also enables us to explore a range of FCA locations to control the flows through the PCAs.

Bear in mind one important assumption the initial route estimation is available. In reality if we run the model, set the rates for the FCA/PCA and ran the TOS allocation algorithm, the new route assignments may not the same as previous route assignment. We may need to run the model again (the convergence issue is dealt in other papers), therefore we say we are solving a subproblem of CTOP FCA/PCA rate optimization.
This use case primarily addresses convective weather activity in southern Washington Center (ZDC) on July 15, 2016. We also assume there is demand-capacity imbalance at EWR airport.

- We assume there is a four-hour capacity reduction in ZDC/EWR from 2000Z to 2359Z.
- A PCA network can be built to model how air traffic traverse through congested regions.
We used historical flight data pulled from September 8, 2016 as a representative clear weather day for traffic demand. SWIM and Coded Departure Route (CDR) database are used to model the demand.

In total 890 flights are captured by this CTOP.

Four time periods are added to make sure all CTOP captured flights can exit the PCA network by the end of the CTOP window.

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</table>

Table 1: Capacity Scenarios
The two-stage solution outperforms the deterministic policy, as it should, since it explicitly considers the uncertainty when making holding decisions.

The semi-dynamic model solution is better than the two-stage model solution and dynamic model in turn performs better than semi-dynamic model, which are also expected, because dynamic models uses more weather information than two-stage static model.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Ground Delay Periods</th>
<th>Air Holding Periods</th>
<th>Total Cost</th>
<th>Expected Cost</th>
<th>Running Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SCEN1</td>
<td>SCEN2</td>
<td>SCEN3</td>
<td>SCEN1</td>
<td>SCEN2</td>
</tr>
<tr>
<td>SCEN1</td>
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<td>88</td>
<td>0</td>
<td>198</td>
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<tr>
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<td>0</td>
<td>0</td>
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<td>470</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>192</td>
<td>192</td>
<td>0</td>
<td>89</td>
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<tr>
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<td>0</td>
</tr>
<tr>
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<td>280</td>
<td>403</td>
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<td>0</td>
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<tr>
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<tr>
<td>Perfect Information</td>
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</tbody>
</table>

Table 2: Deterministic vs. Stochastic Solutions Comparison ($c_a/c_g = 2$)
The optimization models are solved using Gurobi 7.5.2 on a laptop with 2.6 GHz processors and 16 GB RAM. Problem sizes decision variables/constraints: two-stage model 1517/1600; semi-dynamic model 10071/6739; dynamic model 21402/15580. The computation times for all models are quite short.

### Table 3: Deterministic vs. Stochastic Solutions Comparison (\(c_a/c_g = 2\))

<table>
<thead>
<tr>
<th></th>
<th>Ground Delay Periods</th>
<th>Air Holding Periods</th>
<th>Total Cost</th>
<th>Expected Cost</th>
<th>Running Time</th>
</tr>
</thead>
<tbody>
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<td>If This Scenario Occurs:</td>
<td>If This Scenario Occurs:</td>
<td></td>
<td></td>
</tr>
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<td>0</td>
<td>470.0</td>
<td>470.0</td>
<td>≪ 1.0</td>
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<tr>
<td>EEV</td>
<td>192</td>
<td>89</td>
<td>192.0</td>
<td>430.6</td>
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<tr>
<td>Perfect Information</td>
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<td>0</td>
<td>88.0</td>
<td>279.4</td>
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</tr>
</tbody>
</table>

The computation times for all models are quite short.
Conclusions

1. We proposed three stochastic models which generalize the classical GDP rate planning models to the case of having multiple constrained resources.

2. We revealed that why the general multiple constrained resources optimization is in nature a multi-commodity flow problem and discussed what should be the correct boundary conditions.

3. We showed the benefit of allowing flights to dynamically adjust departure times by exploiting weather, departure and en route time information.

4. In the stochastic optimization framework, like Mukherjee’s work gives the theoretic lower bound for the single airport ground hold problem, our dynamic stochastic model gives the theoretic lower bound for the general multiple constrained resources ground and air hold problem.

5. The models are formulated to be aggregate and therefore the problem size does not depend on the number of captured flights. The preliminary computational result is promising.

6. We pointed out these models are closely related with CTOP rate optimization problem and can help air traffic managers implement and perform post-analysis for CTOP programs.
Advice and comments?